

The University of Chicago

PHOTOGRAPHIC DETERMINATION
OF MASS RATIOS OF VISUAL
BINARIES

A PART OF A DISSERTATION SUBMITTED
TO THE FACULTY OF THE DIVISION OF THE
PHYSICAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

1934

By
RALPH CRAIG HUFFER

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By RALPH C. HUFFER

ABSTRACT

By means of plates taken with the 40-inch refractor at epochs separated by fourteen to twenty-eight years, the mass ratios of seventeen visual binaries have been determined. The masses of these and of all others with known mass ratios are computed and plotted on a mass-luminosity diagram. In general there is agreement with the Eddington curve, both in the positions of the points on the chart and in the mass ratios as indicated by the slopes of the lines joining the two components of the pairs.

The values of the mass ratios are plotted as functions of spectral type, period, total mass of the system, and difference of magnitude of the components.

The photographic determination of the mass ratio of a visual binary requires plates of large scale covering a long period of time. During the interval the orbital motion must have been such that the components show measurable departure from uniform rectilinear motion.

In 1916 G. Van Biesbroeck, recognizing the need and value of mass ratios, added a number of binaries to the regular parallax program of the Yerkes Observatory. At that time he outlined¹ the various cases in which a determination could be made and presented four illustrations of the methods of solution.

The test of the theory of A. S. Eddington² on the relationship between stellar masses and luminosities, published in 1924, depended in the region of the dwarf stars on a very limited number of visual binaries. Of these the mass ratios were uncertain or unknown in all but four. The present investigation covers all stars for which suitable plate material is now at hand.

According to the data available, one of three methods was used. The first was in the case of stars of approximately equal magnitude in which the separation was sufficient to insure against the interference of the images. Here the measures alone, corrected for annual parallax, were sufficient for the completion of the solution. The second procedure was used in the cases of stars of unequal magnitude in which only the brighter appeared on the plates. Here wide separa-

¹ *A.J.*, 29, 173, 1916.

² *M.N.*, 84, 308, 1924.

tion was not necessary, but in this case the position of the fainter component had to be computed from a satisfactory orbit, if available. The third method was used when no orbit covered the time of the plates sufficiently well. Then micrometric measures over the entire interval were used as a basis for interpolating the polar co-ordinates of the pair. Plates at three epochs were always available and solutions depending on catalogue values of the proper motions were not necessary.

In two cases it seemed desirable to compute new orbital elements, as existing ones were not in agreement with recent micrometric meas-

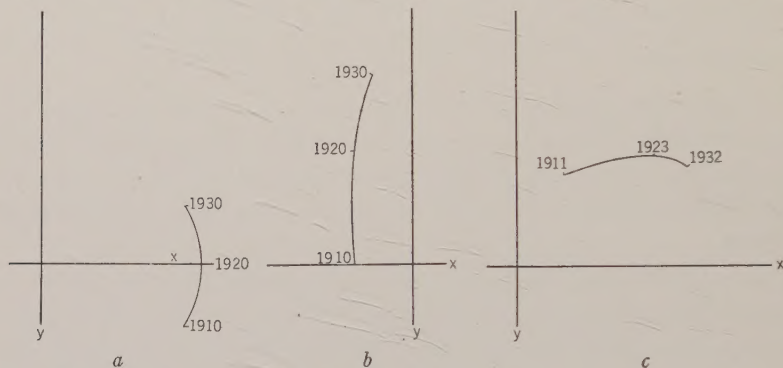


FIG. 1

ures and gave poor determinations of the mass ratios. The following elements for 26 Draconis were computed by Van Biesbroeck in September, 1933, and have been previously unpublished: P , 80.6 years; T , 1873.2; e , 0.10; Ω , $153^{\circ}18'$; ω , $330^{\circ}0'$; i , $73^{\circ}8'$; a , $1''.51$. In October of the same year the writer computed the following elements for β 648: P , 61.8 years; T , 1911.2; e , 0.25; Ω , $48^{\circ}7'$; ω , $281^{\circ}4'$; i , $65^{\circ}0'$; a , $1''.24$. In both pairs the motion is retrograde.

In the majority of stars measures were made in both rectangular co-ordinates x and y , corresponding to the directions of α and δ at the binary. In the case of ζ Herculis, however, the motion was such that the measures in only one direction were justified. As an example of the bases for decision as to the direction of measure, consider a star moving in the apparent orbit as shown in Figure 1a. Here the curvature is such that measures in x would show a considerable de-

viation from uniform motion, while measures in y would indicate practical uniformity. In Figure 1*b* the deviation in y , caused largely by retarded motion, would be about 60 per cent greater than that in x , caused by curvature. In the case of a star moving with an inclination of 90° and node of 0° , the change in x would be zero and hence would add nothing to the solution, while under certain conditions a determination from y -measures, using the deviation from uniform velocity, would be easily possible. Figure 1*c* shows the part of the orbit of 70 Ophiuchi covered by the plates in the present determination. The weight of the solution in x , dependent largely on velocities, is 1.6 times that in y , which uses mainly curvature. The results in the case of this star and of others indicate that the suggestion of H. L. Alden³ that measures should always be made in the direction perpendicular to the arc at the middle point of the apparent orbit needs to be altered to make use of variations in velocity as well as of curvature.

The data and results for seventeen binaries are given in Table I. The plates were measured and reduced by the method used at this observatory in the determination of parallaxes. Conditional equations of the type

$$\mu_x t + k_x - r_x B = m_x$$

were formed, where μ_x is the proper motion in x , k_x a constant, r_x the separation of the two components, t the time at which the plate was made, B the ratio $m_2/(m_1 + m_2)$ or mass of the companion in terms of total mass, and m_x the measured position of the brighter star after correction for parallax. Measures in y gave rise to similar equations. Solutions were usually first made in x and y together. If residuals in this solution indicated disagreement in x or y , separate solutions were made in each direction. Where this was not done, the residuals indicated that better results could not be expected from separate solutions. Results are given in the last two columns of Table I as obtained from least-squares solutions for B from the equations in x , in y , and in x and y together. Some previous determinations are given for each star when such have been made. The sources are des-

³ *Pop. Astr.*, 33, 164, 1925.

TABLE I
DETERMINATIONS OF MASS RATIOS

STAR	APPAR- ENT VISUAL MAGNI- TUDES	PARALLAX IN SECONDS OF ARC; SOURCE OF ORBIT	NO. OF PLATES; INTERVAL (YEARS)	$m_2/(m_1+m_2)$	
				From x ; from y	From x and y ; from Other Sources
η Cas.	3.64	0.180	6		$.24 \pm .01$
$0^h 43^m$	7.9	Doberck	27		VB, .21; M, .25; B, .43
48 Cas.	4.8	0.028	6		$.50 \pm .08$
$1^h 54^m$	7.2	Bennot	22		
Procyon	0.48	0.310	7		$.26 \pm .01$
$7^h 34^m$	13.5	Jones	16		J, .24; B, .25
ϵ Hya AB ...	4.0	0.024	7		$.26 \pm .09$
$8^h 42^m$	5.5	Aitken	14		M, .50; R, .40
ξ Boo.	4.7	0.155	6	$.37 \pm .12$	$.49 \pm .05$
$14^h 47^m$	6.7	None	15	$.51 \pm .06$	B, .47
ι Boo.	5.2	0.078	6		$.54 \pm .14$
$15^h 0^m$	6.0	None	17	$.39 \pm .06$	
γ CrB.	4.0	0.020	6		$.50 \pm .16$
$15^h 39^m$	7.0	None	21		
ζ Her.	3.0	0.106	6		
$16^h 37^m$	6.5	Silbernagel	13	$.51 \pm .04$	B, .30
26 Dra.	5.3	0.062	7		$.47 \pm .05$
$17^h 34^m$	9.3	V. Bies- broeck	21		
70 Oph.	4.1	0.198	7	$.49 \pm .02$	$.50 \pm .03$
$18^h 0^m$	6.1	None	21	$.51 \pm .05$	VB, .47; M, .44; B, .45
99 Her.	5.2	0.053	9		$.30 \pm .13$
$18^h 3^m$	9.7	None	14		K, .04
β 648.	5.2	0.072	7	$.91 \pm .08$	
$18^h 54^m$	8.7	Huffer	14	$.35 \pm .03$	K, .17
61 Cyg.	5.6	0.300	8	$.49 \pm .27$	$.69 \pm .13$
$21^h 2^m$	6.3	None	27	$.77 \pm .09$	
τ Cyg.	3.8	0.051	6		$.40 \pm .04$
$21^h 11^m$	8.0	V. Bies- broeck	22		VB, .47
ζ Aqr.	4.4	0.028	6	$.57 \pm .20$	$.15 \pm .11$
$22^h 24^m$	4.6	None	14	$.22 \pm .09$	
Kr 60.	9.3	0.257	6		$.37 \pm .01$
$22^h 24^m$	10.8	None	27		M, .46; Alden, .46

TABLE I—*Continued*

STAR	APPAR- ENT VISUAL MAGNI- TUDES	PARALLAX IN LEGENDS OF ARC; SOURCE OF ORBIT	NO. OF PLATES; INTERVAL (YEARS)	$m_2/(m_1+m_2)$	
				From x ; from y	From x and y ; from Other Sources
π Cep.	5.2	0.015	6	.28 $\pm$.09	.09 $\pm$.22
23 ^h 5 ^m	7.5	Laurit- zen (II)	14		
85 Peg.	5.8	0.090	7	.31 $\pm$.04	.47 $\pm$.04
23 ^h 57 ^m	11.0	V. Bies- broeck	21	.57 $\pm$.03	B, .64; VB, .64
85 Peg.				.30 $\pm$.07	.42 $\pm$.03
23 ^h 57 ^m		Allen		.48 $\pm$.02	Allen, .64

ignated as follows: B, Lewis Boss;⁴ VB, G. Van Biesbroeck;⁵ M, S. A. Mitchell;⁶ J, H. S. Jones;⁷ and K, P. Van de Kamp.⁸

The relevant facts about all the stars for which mass ratios are known have been collected in Table II. The parallaxes are taken from F. Schlesinger's "Catalogue of Bright Stars"⁹ and "Catalogue of Parallaxes."¹⁰ The absolute magnitudes are computed from apparent magnitudes from the same sources or from R. G. Aitken's "New General Catalogue of Double Stars."¹¹ Masses of the separate components have been assigned on the basis of the mass ratios listed. Six of the stars have given some evidence of multiplicity. The three stars ι Bootis B, γ Coronae Borealis A, and σ Coronae Borealis A are spectroscopic binaries with two spectra that are much alike in structure and intensity. Each has been treated as a single star of half the mass and half the luminosity that would otherwise be assigned to it. The other three— λ Ophiuchi A, τ Cygni A, and π Cephei A—show only single spectra. Because of uncertainty of duplicity, or at least because of a fair assurance that one component is relatively faint, each has been treated as a single star possessing the entire luminosity and mass of the visual component.

⁴ *Preliminary General Catalogue, Carnegie Institution of Washington, 1910.*

⁵ *Op. cit.*

⁶ *Pub. Leander McCormick Obs.*, 3, 661, 1920.

⁷ *M.N.*, 88, 432, 1928.

¹¹ *Carnegie Institute of Washington, 1932.*

⁸ *Pop. Astr.*, 38, 478, 1930.

⁹ *Pub. Yale Univ. Obs.*, 1930.

¹⁰ *Ibid.*, 1924.

TABLE II
DATA ON VISUAL BINARIES WITH KNOWN MASS RATIOS

Star ADS No.	Paral- lax	Abs. Vis. Mags.	$m_2/(m_1+m_2)$	Spec- tra	Computer of Orbit and of Mass Ratio	a'' P (Yrs.)	Masses (Sun=1)
η Cas.....	0.180	4.9	.24	F8	Doberck	12.21	.91
671.....		9.2		M1	Huffer	508	.30
48 Cas.....	.028	2.0	.50	A3	Bennot	0.66	1.62
1598.....		4.4			Huffer	63.3	1.65
σ Eri BC.....	.203	10.7	.31	A2n	V. d. Bos	6.89	.44
3093.....		12.7		M6e	V. d. Bos	248	.20
80 Tau.....	.021	3.6	.31	F0	V. d. Bos	1.04	3.79
3264.....		6.1			V. d. Bos	148.3	1.67
α Aur.....	.075	0.2	.44	gGo	Merrill	0.054	4.20
3841.....		0.2			Merrill	0.285	3.30
α CMa.....	.376	1.3	.32	A2s	Aitken	7.57	2.22
5423.....		11.3		F	Auwers	50.04	1.04
α Gem.....	.077	2.1	.44	A2s	Doberck	6.57	1.53
6175.....		3.1		A2s	Russell	477	1.20
α CMi.....	.310	3.0	.26	∓ 5	Jones	4.26	1.20
6251.....		16.0			Huffer	40.2	.41
ϵ Hya AB.....	.024	0.6	.26	gF9	Aitken	0.23	2.79
6993.....		2.1		F5	Huffer	15.3	.97
ξ UMa.....	.146	4.8	.32	G0	V. d. Bos	2.54	1.37
8119.....		5.7			V. d. Bos	59.9	.63
γ Vir.....	.073	2.3	.50	F0	Lohse	3.62	1.30
8627.....		2.3		F0	Boss	178	1.30
α Cen.....	.760	4.7	.46	G0	Finsen	17.66	1.06
		6.1		K5	Alden	80.1	.92
ξ Boo.....	.155	5.7	.49	G5	Doberck	4.87	.70
9413.....		7.7		K5	Huffer	151	.66
ι Boo.....	.078	4.7	.54	G0	Doberck	3.58	1.05
9494.....		6.3		G2p	Huffer	205	.62
γ CrB.....	.020	1.3	.50	A0	Comstock	0.62	.72
9757.....		3.5			Huffer	101	1.47
σ CrB.....	.048	4.8	.32	G0	Doberck	6.00	.85
9979.....		5.1			Boss	900	.80
λ Oph.....	0.028	1.2	.50	A0	Rabe.....	0.94	1.04
10087.....		3.3			Boss	135.3	1.04

TABLE II—*Continued*

Star ADS No.	Paral- lax	Abs. Vis. Mags.	$m_2/(m_1+m_2)$	Spec- tra	Computer of Orbit and of Mass Ratio	a'' P (Yrs.)	Masses (Sun=1)
ζ Her.....	0.106	3.1	.51	Go	Silbernagel	1.35	.86
10157.....		6.6			Huffer	34.5	.88
β 416.....	.143	7.2	.43	K2	Voute	1.83	.4
		9.2			Voute	42.2	.3
26 Dra.....	.062	1.3	.47	F8	V. Biesbroeck	1.51	1.18
10660.....		5.3			Huffer	80.6	1.04
μ Her BC.....	.111	7.1	.50	M5	Aitken	1.30	.44
10786.....		10.3			Hertzprung	43.23	.44
70 Oph.....	.198	5.6	.50	K1	Pavel	4.50	.67
11046.....		7.6		K5	Huffer	88	.67
99 Her.....	.053	3.8	.30	F8	Makemson	1.03	1.64
11077.....		8.3			Huffer	56	.70
β 648.....	.072	4.5	.35	Go	Huffer	1.24	.87
11871.....		8.0			Huffer	61.8	.47
61 Cyg.....	.300	8.0	.69	K8	Baise	32.8	.71
14636.....		8.7		M0	Huffer	756	1.58
τ Cyg.....	.051	2.3	.40	F0	Aitken	0.96	1.64
14787.....		6.5			Huffer	49.2	1.12
κ Peg.....	.025	1.9	.36	F1s	Luyten	0.22	3.3
15281.....		2.4		K0	Luyten	11.52	1.9
ζ Aqr.....	.028	1.6	.57	F2s			
15971.....		1.8		F1s	Huffer		
Kr 60.....	.257	11.3	.37	M3	Huffer	2.36	.25
15972.....		12.8		M4	Huffer	44.5	.14
π Cep.....	.015	1.1	.28	gG5	Lauritzen	0.92	5.20
16538.....		3.4			Huffer	178	2.07
85 Peg.....	0.090	5.6	.45	Go	V. Biesbroeck	0.82	.60
17175.....		12.8			Huffer	26.3	.49

In Figure 2 the absolute visual magnitudes have been plotted as ordinates against the logarithms of the masses. The Eddington mass-luminosity curve, adjusted to visual magnitudes, has also been drawn. While the scatter is considerable, as is to be expected from the uncertain orbits, parallaxes, and mass ratios entering into the solution for mass, there is, however, a very definite clustering about

the curve and the lines joining the pairs in the main tend to be parallel to the curve.

Figure 2 was originally plotted, using bolometric magnitudes as in Eddington's original article. The conversion from visual to bolometric magnitudes depends, however, on spectral type, and since for most of the faint components of the binaries in Table II the types are unknown or uncertain, it seemed wiser to use the visual magni-

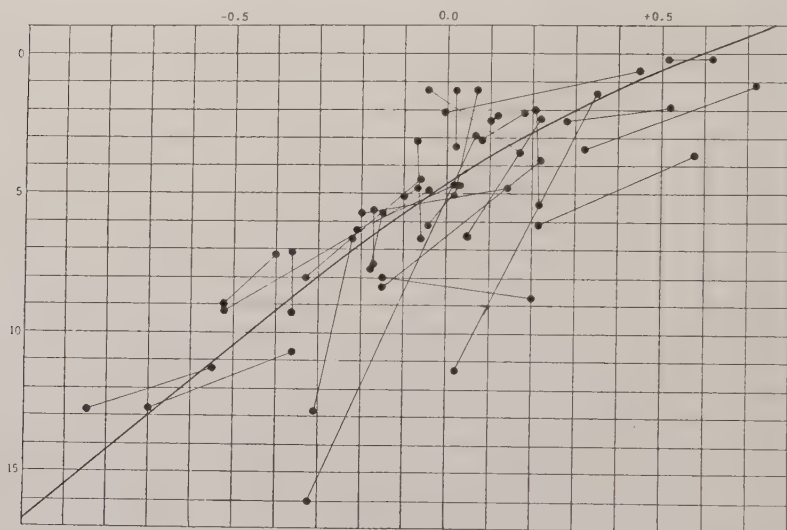


FIG. 2.—The ordinates are absolute magnitudes and the abscissas are logarithms of masses. The curve is Eddington's mass-luminosity law. The straight lines join the two components of a binary.

tudes in preference to reducing the number of stars. While this procedure might introduce serious difficulties into an analytic discussion of the problem, it makes apparently little difference in the agreement with reference to the graph.

In Table III the logarithms of the observed masses are compared with the values as read from the curve of mass luminosity, and the differences are listed. Of the sixty stars plotted, the twelve which show a disagreement of more than 0.25 in the logarithms of the masses are considered individually in the following paragraphs.

Both components of 80 Tauri lie below the mass-luminosity curve in Figure 2, although the line joining them is essentially parallel to

the curve. The chief weakness here is in the parallax. Aitken gives the parallax as $0''.011$ as obtained trigonometrically, and calls it "very uncertain." The cluster value is $0''.026$, which is also the value

TABLE III
COMPARISON OF COMPUTED MASSES WITH VALUES FROM
THE MASS-LUMINOSITY CURVE

STAR	LOG MASS OF PRIMARY			LOG MASS OF COMPANION		
	Observed	Computed	O-C	Observed	Computed	O-C
η Cas.....	-0.04	-0.02	-0.02	-0.52	-0.40	-0.12
48 Cas.....	+ .21	+ .31	- .10	+ .22	+ .04	+ .18
α_2 Eri.....	- .36	- .52	+ .16	- .52	- .68	+ .16
80 Tau.....	+ .58	+ .10	+ .48	+ .22	- .14	+ .36
α Aur.....	+ .62	+ .57	+ .05	+ .52	+ .57	- .05
α CMa.....	+ .35	+ .39	- .04	+ .02	- .57	+ .59
α Gem.....	+ .18	+ .30	- .12	- .08	+ .18	- .10
α CMi.....	+ .08	+ .20	- .12	- .39	- .94	+ .55
ϵ Hya.....	+ .45	+ .51	- .06	- .01	+ .30	- .31
ξ UMa.....	+ .14	.00	+ .14	- .20	- .10	- .10
γ Vir.....	+ .11	+ .27	- .16	+ .11	+ .28	- .17
α Cen.....	+ .04	+ .01	+ .03	- .03	- .14	+ .11
ξ Boo.....	- .16	- .10	- .06	- .18	- .28	+ .10
ι Boo.....	+ .03	+ .01	+ .02	- .21	- .16	- .05
γ CrB.....	- .14	+ .40	- .54	+ .17	+ .27	- .10
σ CrB.....	- .07	.00	- .07	- .10	- .03	- .07
λ Oph.....	+ .02	+ .41	- .39	+ .02	+ .16	- .14
ζ Her.....	- .07	+ .19	- .26	- .06	- .19	+ .13
β 416.....	- .40	- .23	- .17	- .52	- .40	- .12
26 Dra.....	+ .07	+ .40	- .33	+ .02	- .06	+ .08
μ Her BC...	- .36	- .22	- .14	- .36	- .47	+ .11
70 Oph.....	- .17	- .09	- .08	- .17	- .27	+ .10
99 Her.....	+ .22	+ .10	+ .12	- .16	- .33	+ .17
β 648.....	- .06	+ .03	- .09	- .33	- .30	- .03
61 Cyg.....	- .15	- .30	+ .15	+ .20	- .36	+ .56
τ Cyg.....	+ .22	+ .28	- .06	+ .05	- .18	+ .23
κ Peg.....	+ .52	+ .32	+ .20	+ .28	+ .26	+ .02
Kr 60.....	- .55	- .56	+ .01	- .85	- .64	- .21
π Cep.....	+ .72	+ .43	+ .29	+ .32	+ .14	+ .18
85 Peg.....	-0.22	-0.09	-0.13	-0.31	-0.64	+0.33

given in Schlesinger's "Catalogue of Bright Stars." An adopted value of $0''.029$ would give points very near the curve.

The well-known white dwarf α Canis Majoris B, with its uncertain magnitude and probable high density, would hardly be expected to lie near the mass-luminosity curve. It is about 7 mag. fainter than its mass would indicate.

Like the companion of Sirius, α Canis Minoris B is probably a white dwarf, is of uncertain magnitude, and is about 7 mag. fainter than the curve would predict on the basis of its observed mass.

ϵ Hydrae B, with an orbit of mean distance $0''.23$, a parallax uncertain between $0''.015$ and $0''.060$, and a mass ratio of $0.26 \pm .09$, is not farther from the curve than might be anticipated.

The spectroscopic binary γ Coronae Borealis A lies far off the curve. Its small parallax of $0''.022$ and its very uncertain mass ratio, as well as the uncertainty of the semi-major axis of the orbit, point to a very low weight of the observed value of the mass.

The trigonometric parallax of λ Ophiuchi A is given by Schlesinger as $0''.000$, although he adopts $0''.028$ in the "Catalogue of Bright Stars." The orbit is still very uncertain and the mass ratio is doubtful. In addition, the radial velocity is variable, introducing possible duplicity into the problem. A parallax of $0''.022$ would bring it into fair agreement.

ζ Herculis A lies above the curve, while its companion lies below, about half as far. If instead of the mass ratio of 0.51 of the present paper the value of 0.30 as given by Boss is adopted, both components will lie slightly above the curve with a difference of about one-tenth between observed and computed logs of the masses.

The orbit of 26 Draconis, because of the difficulty of measuring with the micrometer a close pair, of magnitudes 5 and 10, is the weakest part of its mass determination. Both the period and the semimajor axis are uncertain at the present time.

The slow-moving pair 61 Cygni has covered so little of its orbit that its period, mean distance, and mass ratio are little more than guesses. It is included in this paper as an interesting test of the method rather than as a reliable determination of a mass ratio.

The parallax of π Cephei is uncertain according to Aitken, who gives the trigonometric value as $0''.000$ and the spectroscopic as $0''.011$ or $0''.026$. Since the line joining the two components is nearly parallel to the curve, it appears that an adopted parallax of $0''.018$ would bring the star into agreement with the Eddington theory.

The faint member of the unusual system 85 Pegasi has repeatedly been shown to be the heavier of the two in various determinations of mass ratio. The results of the present paper from a number of dif-

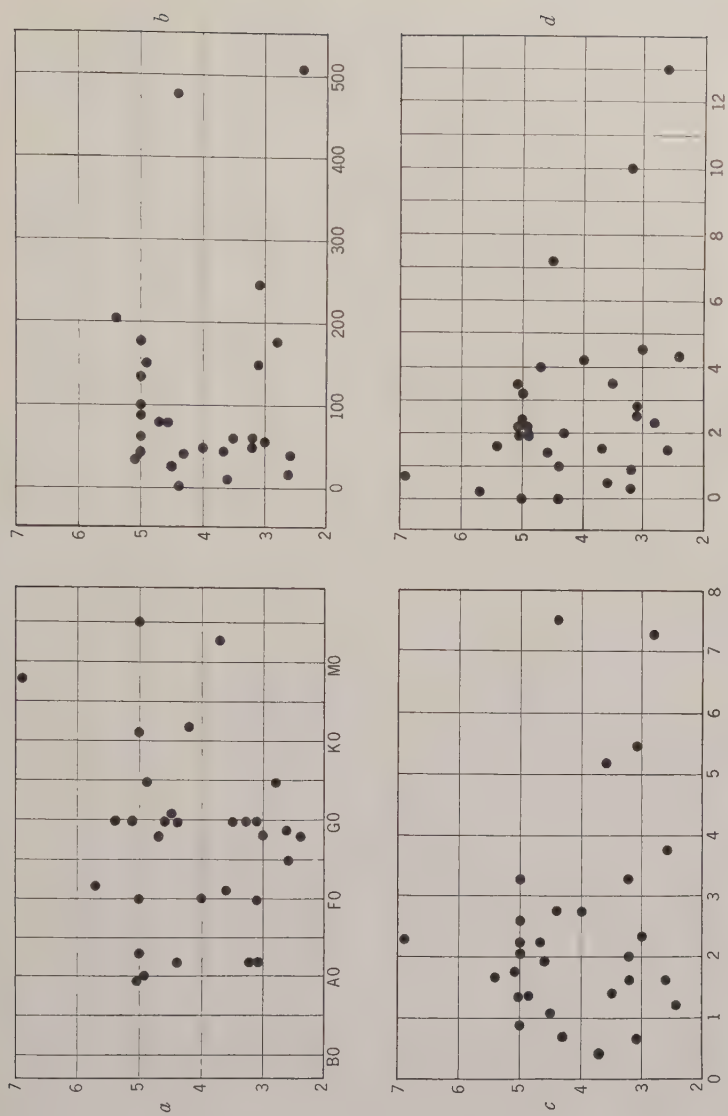


FIG. 3.—Mass ratio as a function (*a*) of spectral type; (*b*) of period in years; (*c*) of total mass of the system; and (*d*) of difference in magnitudes of the components.

ferent analyses are not conclusive. Since the companion is about 5 mag. fainter than its mass would indicate that it should be, and since its spectrum is not available, the suggestion that this star of absolute magnitude 12.8 is a white dwarf may not be out of place.

Of the sixty stars on the diagram in Figure 2, the remaining forty-eight show fair agreement with the Eddington theory. Of the twelve aberrant ones, six have uncertain parallaxes of less than $0''.028$, three are suspected white dwarfs, one can be brought into agreement by adoption of Boss's value of the mass ratio, and the remaining two are stars of very uncertain orbital elements. With the exception of the three suspected white dwarfs, it appears that there is no star on the list which can be said to be sharply in disagreement with the Eddington theory.

The thirty-one mass ratios of visual binaries given in Table II may throw some light on the changes in the relative masses of these systems during their life-histories and may make some contribution to the explanation of their origin. With this in mind, Figure 3 has been prepared in which the mass ratio is plotted as a function of various variables which might be expected to undergo systematic changes in the period of existence of the star.

Figure 3*a* shows the mass ratio plotted against the spectral type of the primary star. No correlation is evident. In Figure 3*b* a graph of mass ratio against period shows a wide scatter in the stars of short period, while the data on the slower binaries are too uncertain and too sparse to justify any conclusions. In Figure 3*c* the scatter of points on the graph of mass ratio against the total mass of the system is obvious. In Figure 3*d* mass ratio is shown as a function of the difference in magnitude between the components of the binary. It shows a tendency for the masses to become more unequal with increasing differences in magnitude, as is indicated by the curve of mass luminosity. The scatter is large, however, for any given magnitude difference, which is indicative of differing relations between these quantities in stars of different luminosities.

The writer wishes to acknowledge his obligation to Dr. Edwin B. Frost and Dr. Otto Struve for their kindness in making the equipment of the observatory available to him and to Dr. G. W. Moffitt

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YERKES OBSERVATORY

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